



**Re:** One-Percent-Plus Flow Frequency Analysis

# **1.0 BACKGROUND**

The Mile High Flood District (MHFD) retained Wright Water Engineers, Inc. (WWE) to develop a procedure and tool for calculating the one-percent-plus (1% plus) flow, in accordance with guidance from the Federal Emergency Management Agency (FEMA). The 1% plus flood elevation is defined by FEMA as "a flood elevation derived by using discharges at the upper 84-percent confidence limit for the 1-percent-annual-chance flood" (FEMA, 2019). WWE has developed a spreadsheet which uses the 2-year, 10-year, and 100-year flows to calculate an upper 84% confidence limit of the 100 year flow (or 1% plus flow), based on FEMA guidance (2019) and equations provided in the United States Geological Survey's (USGS) Bulletin 17C (England et al., 2019) and Bulletin 17B (Interagency Advisory Committee on Water Data, 1981). Dr. James Guo, P.E., provided peer review of WWE's methodology and spreadsheet tool. A memorandum provided by Dr. Guo summarizing his review is attached as Attachment A. Dr. Guo is an expert in statistical and stochastic hydrologic analyses, among other topics. Dr. Jayantha Obeysekera, P.E., of Florida International University also provided peer review of WWE's methodology and spreadsheet tool. Dr. Obeysekera is an expert in stochastic hydrology who consults with WWE as an adjunct scientist. This methodology was also reviewed by the STARR II team for FEMA and is consistent with FEMA's Guidance and Standards. The corresponding STARR II memo is included as Attachment B.

This memorandum provides documentation and discussion on the underlying analytical process for calculating the 1% plus flow as well as evaluation of the spreadsheet using case-studies from nine gages in the Denver metropolitan area. WWE's spreadsheet (attached) is based on the FEMA and Bulletin 17B guidance and generally matches well with the 1% plus calculations computed using real gage data.

# <span id="page-0-0"></span>**2.0 FEMA, USACE AND USGS BULLETIN 17C AND 17B GUIDANCE**

# **2.1 Calculation Methodology**

In developing the 1% plus spreadsheet, WWE reviewed documentation from FEMA, the United States Army Corps of Engineers (USACE), and USGS.

FEMA (2019) provides the following discussion of the 1% plus flow and calculation methodology:

*The 1-percent-plus flood elevation for a study utilizing rainfall-runoff methodology is defined as a flood elevation derived by using discharges at the upper 84-percent confidence limit for the 1-percent-annual-chance flood. 1-percent + discharges can be estimated using methods outlined in Bulletin 17C appendix 7 (Expected Moments Algorithm), and Chapter 4 of the USACE document Risk-Based Analysis for Flood Damage Reduction Studies (EM 1110-2-1619, USACE, 1996). Equations in Appendix 5 are used to determine synthetic logarithmic skew coefficients, standard deviation, and mean. These values paired with equivalent record length of the rainfall-runoff model estimated based on methods shown in Table 4-5 of Chapter 4 of EM 110-2-1619, are used in equations in Appendix 7 of Bulletin 17C to calculate the upper confidence limit discharge. The equivalent record length of the rainfallrunoff model is estimated based on the source data and the amount of detail and calibration that was provided with the model inputs as outlined in Table 4-5 of Chapter 4 of the USACE document Risk-Based Analysis for Flood Damage Reduction Studies (EM 1110-2-1619, USACE, 1996).*

However, upon review of both USGS Bulletin 17B and Bulletin 17C (England et al., 2019), WWE determined that while some relevant equations are found in Bulletin 17C and USACE EM 1110-2- 1619, some of the relevant equations must be found in Bulletin 17B (note that the above quotation references equations in Appendix 5, which is Appendix 5 of Bulletin 17B). Where possible, WWE has referenced equations in Bulletin 17C, however some equations are not included in that document and are from Bulletin 17B or USACE EM 1110-2-1619. The calculation methodology is described below and is based on using the modeled 2-year, 10-year, and 100-year flows to calculate synthetic statistics that are used to determine the shape of the flow frequency curve and associated confidence limits.

First, a synthetic logarithmic skew coefficient  $(G<sub>S</sub>)$  is calculated using the 2-year  $(Q<sub>0.50</sub>)$ , 10-year  $(Q<sub>0.10</sub>)$ , and 100-year flows  $(Q<sub>0.01</sub>)$ , based on Equation 5-3 of Bulletin 17B, given below:

$$
G_S = -2.50 + 3.12 \frac{Log(Q_{0.01}/Q_{0.10})}{Log(Q_{0.10}/Q_{0.50})}
$$

Bulletin 17B notes that Equation 5-3 is an approximation appropriate for use between skew values of +2.5 and -2.0. Skew values outside this range are discussed in greater detail in Section [3.4.](#page-11-0) The skew coefficient relates to the shape, or steepness, of the flow frequency curve.

The skew coefficient is then used to look up Log Pearson III deviates for exceedance probabilities of 0.01 and 0.50 ( $K<sub>Gs,0.01</sub>$  and  $K<sub>Gs,0.50</sub>$ , respectively). The lookup tables, which use the calculated skew and the exceedance probability of interest are found in Bulletin 17B, Appendix 3. Note that the spreadsheet WWE developed includes a lookup table of the values printed in Appendix 3. Linear interpolation is used to calculate the K values that reflect a generalized skew value calculated to the hundredths place (Appendix 3 provides lookup tables based on skew values to the tenth place).

These Log Pearson III deviates ( $K_{Gs,0.01}$  and  $K_{Gs,0.50}$ ), in combination with the 2-year and 100-year flows, are used to calculate additional synthetic statistics: the standard deviation  $(S_S)$  and mean  $(X_S)$ . The synthetic standard deviation and mean are calculated according to the following equations (Equations 5-4 and 5-5 in Bulletin 17B, respectively):

$$
S_S = \frac{Log(Q_{0.01}/Q_{0.50})}{K_{0.01} - K_{0.50}}
$$

$$
X_S = Log(Q_{0.50}) - K_{0.50}(S_S)
$$

Finally, the above results are used to calculate associated confidence limits. Appendix 9 of Bulletin 17B provides the following equation (Equation 9-3a) for calculating the upper confidence limit ( $U_{P,C}$ ). This equation is also provided as Equation 7-31 and 7-32 in Bulletin 17C, with different variable notation, as well as Chapter 4 of USACE EM 1110-2-1619. For the 1% plus flow frequency, the confidence limit of interest is the upper 84% confidence limit associated with the 1% chance flow, in other words  $U_{0.01, 0.84}$ :

$$
U_{0.01,0.84} = X_S + S_S(K_{0.01,0.84}^U)
$$

Where  $X<sub>S</sub>$  is the synthetic mean calculated previously, and  $S<sub>S</sub>$  is the synthetic standard deviation calculated previously.  $K^{\text{U}}_{0.01,0.84}$  is calculated based on Equation 9-4a of Bulletin 17B as well as in Chapter 4 of USACE EM 1110-2-1619, below:

$$
K_{0.01,0.84}^U = \frac{K_{Gs,0.01} + \sqrt{(K_{GS,0.01})^2 - ab}}{a}
$$

In which:

 $K_{\text{Gs, 0.01}}$  is the value found in the Bulletin 17B lookup tables (discussed above)

$$
a = 1 - \frac{(Z_{0.84})^2}{2(N-1)}
$$
  

$$
b = (K_{GS,0.01})^2 - \frac{(Z_{0.84})^2}{N}
$$

 $Z_{0.84} = 1$  (standard normal deviate)

N is the assumed period of record, which is selected to reflect the confidence in the accuracy of the 2-yr, 10-yr, and 100-yr flows. The USACE's Risk-Based Analysis for Flood Damage Reduction Studies (EM 1110-2-1619, USACE, 1996) provides guidance on selecting N values. USACE recommends that for flow frequency values estimated using a rainfall-runoff-

> routing model with regional model parameters (no rainfall-runoff-routing model calibration) an equivalent record length of 10 to 30 years be used. This is "based on judgment to account for the quality of any data used in the analysis, for the degree of confidence in models, and for previous experience with similar studies." More discussion of selecting an appropriate assumed period of record, as it applies to use for the MHFD, is discussed in Section [3.3.](#page-10-0)

The final 1% plus flow is calculated to be 10 to the power of  $U_{0.01,0.84}$ :

 $1\%$  Plus Flow =  $10^{U_{0.01,0.84}}$ 

WWE input these equations into the attached spreadsheet. The required user inputs are the 2-year, 10 year, and 100-year flows, the assumed period of record, as well as  $K<sub>Gs,0.01</sub>$  and  $K<sub>Gs,0.50</sub>$  from the Bulletin 17B lookup tables. The rest of the spreadsheet calculates the intermediate values provided above and ultimately the 1% plus flow.

Section [2.2](#page-3-0) provides an example of these calculations using example 2-year, 10-year and 100-year flows.

Section [3.0](#page-5-0) provides an evaluation of the reasonableness of these equations and calculated 1% plus flow based on a comparison with previous flow frequency analysis computed for nine example locations in the Denver metropolitan area.

#### <span id="page-3-0"></span>**2.2 Example Calculation**

An example 1% plus calculation using the above described equations is provided below:

**Assume that the 2-year flow = 50 cfs, 10-year flow = 300 cfs, and 100-year flow = 1,500 cfs.** 

Then, 
$$
G_S = -2.50 + 3.12 \frac{Log(Q_{0.01}/Q_{0.10})}{Log(Q_{0.10}/Q_{0.50})}
$$
  
 $G_S = -2.50 + 3.12 \frac{Log(1,500/300)}{Log(300/50)} = 0.30$ 

Returning to the Bulletin 17B lookup tables in Appendix 3 for the Log Pearson III deviates  $(K_{\text{Gs, 0.01}}$  and  $K_{\text{Gx, 0.50}}$ , the values associated with  $G = 0.30$  and probabilities of 0.01 and 0.50, respectively, are:

 $K_{Gs, 0.01} = 2.54421$  $K_{Gs. 0.50} = -0.04993$ 

Using these values to calculate the remaining synthetic statistics yields:

$$
S_S = \frac{Log(Q_{0.01}/Q_{0.50})}{K_{0.01} - K_{0.50}}
$$
  
\n
$$
S_S = \frac{Log(1500/50)}{2.54421 - (-0.04993)} = 0.57
$$
  
\n
$$
X_S = Log(Q_{0.50}) - K_{0.50}(S_S)
$$
  
\n
$$
X_S = Log(50) - (-0.04993)(0.57) = 1.73
$$

Turning to the next set of equations, and selecting an assumed period of record (N value) of 30 years,

$$
a = 1 - \frac{(Z_{0.84})^2}{2(N-1)}
$$
  
\n
$$
a = 1 - \frac{(1)^2}{2(30-1)} = 0.98
$$
  
\n
$$
b = (K_{GS,0.01})^2 - \frac{(Z_{0.84})^2}{N}
$$
  
\n
$$
b = (2.54421)^2 - \frac{(1)^2}{30} = 6.44
$$
  
\n
$$
K_{0.01,0.84}^U = \frac{K_{GS,0.01} + \sqrt{(K_{GS,0.01})^2 - ab}}{a}
$$
  
\n
$$
K_{0.01,0.84}^U = \frac{2.54421 + \sqrt{(2.54421)^2 - (0.98)(6.44)}}{(0.98)} = 3.01
$$
  
\n
$$
U_{0.01,0.84} = X_S + S_S(K_{0.01,0.84}^U)
$$
  
\n
$$
U_{0.01,0.84} = 1.73 + 0.57(3.01) = 3.45
$$

 $1\%$  Plus Flow =  $10^{U_{0.01,0.84}}$ 

1% *Plus Flow* =  $10^{3.45}$  = 2818 cfs

Note that this calculated value varies slightly from the value calculated using WWE's spreadsheet, due to the rounding error introduced in the example calculations as they are written out above.

## **2.3 Note on 2-D Models and unsteady 1-D Models**

Note that the above methodology is specific for use with 1-D steady flow HEC-RAS models. If a 2- D model or unsteady 1-D model is being used for mapping, a different approach may be necessary. In those cases, the 1% plus flow should be calculated by making an adjustment to the modeled rainfall value instead of the runoff value (as outlined in the previous sections). A methodology for estimating the 1% plus flood hydrograph from the 1% plus frequency storm is included in Attachment B (STARR II review of MHFD 1% plus methodology for FEMA).

# <span id="page-5-0"></span>**3.0 SPREADSHEET/EQUATION EVALUATION**

In order to evaluate the reasonableness of the above equations and developed spreadsheet, WWE conducted an evaluation to compare the results calculated using the above equations with results using a full gage-record flow frequency analysis as well as to analyze various sensitivity factors. Discussion of this evaluation is provided in the following sections.

# <span id="page-5-1"></span>**3.1 Gage Analysis versus Spreadsheet Calculation Comparison**

In order to evaluate the reasonableness of the calculation methodology, WWE compared the results of the 1% plus flow calculation using the synthetic statistics equations described above with the results computed using a full gage record. WWE had previously used HEC-SSP and Bulletin 17C to analyze nine gages in the Denver metropolitan area for MHFD, so these locations were selected as the case studies for this evaluation. WWE used the 2-year, 10-year, and 100-year flows that were calculated in HEC-SSP and input them into the 1% plus spreadsheet (with formulas described in Section [2.0\)](#page-0-0). This represents a scenario where the hydrologic modeling exactly matches with the real-world gage data. WWE used an assumed period of record (N value) of 30 years for this initial evaluation (USACE recommends 10-30 years). Separately, WWE used HEC-SSP to calculate the upper 84% confidence limit based on the full gage record and Bulletin 17C procedure. A comparison of these results is presented in [Table 1.](#page-6-0)

A review of the results demonstrates that (assuming accurate 2-year, 10-year, and 100-year flows), the calculated upper confidence limit based only on the 2-year, 10-year, and 100-year flows is generally similar to the confidence limit that would be calculated by analyzing a full gage record. The percent difference between the 1% plus flow calculated with the spreadsheet and synthetic statistics versus the 1% plus flow calculated using the full gage record and a 17C analysis in HEC-SSP ranged from -1% to -32% (a negative percent difference represents that the flow calculated using the synthetic statistics/spreadsheet was less than the flow calculated using the full gage record and Bulletin 17C). The synthetic statistics were consistently lower than the full gage analysis but varied by location.

[Table 2](#page-7-0) presents a similar comparison, except instead of using an assumed period of record of 30 years, the assumed period of record was set equal to the gage record used for the HEC-SSP analysis. WWE notes that in general, there was greater agreement between the synthetic statistics/spreadsheet method and the HEC-SSP 17C method when an assumed period of record equal to the gage record was used (although that is not true in all cases). This is not surprising, because the Bulletin 17C method calculates larger uncertainty when there are fewer years of gage data, and updating the 1% plus synthetic statistics inputs to match the shorter period of record would have a similar result. An assumed period of record of 30 years was used in [Table 1](#page-6-0) to evaluate the suggested methodology when CUHP rainfall-runoff model is used. In most cases when the 1% plus methodology is used, there will be no gage to compare directly against, and thus the selection of the assumed period of record length as done i[n Table 2](#page-7-0) would not be relevant for most studies. Changing the assumed period of record is valuable in this case as a comparison of the accuracy of the 1% plus synthetic statistics methodology compared with Bulletin 17C analysis.

<span id="page-6-0"></span>



<sup>1</sup> Percent Difference uses the 1% plus flow calculated using Bulletin 17C and the full gage record as the baseline value. A negative percent difference indicates that the 1% plus flow calculated using synthetic statistics/spreadsheet is less than the 1% plus flow calculated using Bulletin 17C and the full gage record, and a positive percent difference indicates that the 1% plus flow calculated using synthetic statistics/spreadsheet is greater than the 1% plus flow calculated using Bulletin 17C and the full gage record.



## <span id="page-7-0"></span>**Table 2. Comparison of 1% Plus Flows Calculated Using Synthetic Statistics Equations and Bulletin 17C Full Gage Record Analysis – Variable Assumed Period of Record**

<sup>1</sup> Percent Difference uses the 1% plus flow calculated using Bulletin 17C and the full gage record as the baseline value. A negative percent difference indicates that the 1% plus flow calculated using synthetic statistics/spreadsheet is less than the 1% plus flow calculated using Bulletin 17C and the full gage record, and a positive percent difference indicates that the 1% plus flow calculated using synthetic statistics/spreadsheet is greater than the 1% plus flow calculated using Bulletin 17C and the full gage record.

A comparison of the calculated 1% plus flow with the 200-year and 500-year flows calculated in HEC-SSP based on the Bulletin 17C full gage analysis is presented in [Table 3.](#page-8-0) WWE notes that in all cases, the 1% plus flow exceeds the 200-year flow, and in one case even slightly exceeds and 500 year flow. The percentage increase from the 100-year flow to the 1% plus flow ranged from 19% to 74%.

<span id="page-8-0"></span>



Overall, this evaluation showed that the synthetic statistic equations used to calculate the upper confidence limit produce generally similar results to those produced when a full gage record can be statistically analyzed to calculate upper 84% confidence limits. In most locations, a full gage record is not available to be statistically analyzed and thus the need to use the synthetic statistics equations that are the focus of this memorandum. The upper 84% confidence limits also tend to be relatively large, exceeding the 200-year flows and in some cases the 500-year flows in the case study locations examined by WWE.

# <span id="page-8-1"></span>**3.2 Modeled Flows Used in Spreadsheet Calculation**

WWE next evaluated the reasonableness of the 1% plus calculated values when 2-year, 10-year, and 100-year flows based on hydrologic modeling were used. WWE notes that the modeled 100-year flows are in many cases higher than the 100-year flows calculated using the gage record. This is a reflection of modeling methodology; for example, inadvertent storage in the watershed is not included in the modeled hydrology. The results of these example calculations are shown in [Table 4.](#page-9-0)

<span id="page-9-0"></span>



Similar to the examples given in Section [3.1,](#page-5-1) at some locations the calculated 1% plus flow exceeds the 500-year modeled flows. The percent increase from the 100-year flow to the 1% plus flow ranged from 24% to 455%. Excluding the high outlier of the Lena Gulch at Maple Grove Reservoir location, the percent increases ranged from 24% to 73%.

The calculated 1% plus flow for the Lena Gulch at Maple Grove Reservoir location is exceptionally high compared to the 100-year and 500-year modeled flows. This is caused by a large difference between the modeled 2-year flow (1 cfs) and the modeled 100-year flow (1,920 cfs) which results in a very large synthetic standard deviation and thus large confidence limits.

WWE also conducted an evaluation of whether using 2-year or 2-year and 10-year flows calculated through statistical gage analysis and used in combination with the modeled 100-year flow would produce more accurate results but in general results were not improved. At this time, WWE does not recommend substituting gage analysis for the lower return frequency events if modeled flows are being used for the large events because it may unreasonably and inconsistently change the shape of the curve and associated synthetic statistics.

## <span id="page-10-0"></span>**3.3 Choice of Assumed Period of Record**

An important judgement decision that is made by the user is the assumed period of record. While USACE (1996) provides guidance that for a rainfall-runoff-routing model with regional model parameters, a value between 10 and 30 years should be used, this is a wide range to select within (see [Table 5\)](#page-10-1). In the previous evaluations discussed, WWE had used an assumed period of record of 30 years. Given the use and calibration of the CUHP since the early 1970's, and the associated relative confidence in the methodology, WWE chose this as a reasonable value for the above evaluations. However, WWE also conducted a sensitivity analysis to compare the results computed using a lower assumed period of record of 20 years. The calculations were done using the modeled 2-year, 10-year, and 100-year flows discussed in Sectio[n 3.2.](#page-8-1) The results of this analysis are presented i[n Table 6.](#page-11-1) The ratio of the 1% plus flow calculated with a 20 year assumed period of record (higher resulting flow) to the 1% plus flow calculated with a 30 year assumed period of record (lower resulting flow) ranged from 107% to 135%.

<span id="page-10-1"></span>

#### **Table 5. Equivalent Record Length Recommendations from USACE (1996)**

<span id="page-11-1"></span>



Lower N values (shorter assumed period of record) increase the calculated 1% plus flow, but the degree to which this occurs also depends on the calculated synthetic statistics (as reflected in the variability in [Table 6\)](#page-11-1). Based on the calibration and long period of use of the CUHP methodology, WWE recommends that an assumed period of record of 30 years be used. However, a lower N value may be more appropriate for use in areas outside of MHFD or for watersheds that do not fit the assumptions of the CUHP cascading-plain approach or where confidence in model results is lower.

# <span id="page-11-0"></span>**3.4 Consideration of Extreme Skew Coefficients**

When calculating the synthetic skew coefficient, Bulletin 17B advises that the equation is appropriate for use between skew values of +2.5 and -2.0. For all of the above examples aside from one analyzed by WWE, the skew fell within this range. The Lena Gulch at Highway 6 calculated skew was 2.51, or just outside the acceptable range. Very high or very low skew values are indicative of curves with a more extreme shape. A frequency curve with a high positive skew typically will have a convex shape that is steeper at the lower-frequency end of the curve. The high positive skew values occur when the ratio of the 100-year to 10-year flows is much greater than the ratio of 10-year to 2-year flows. This may be indicative of modeling methodology that overestimates either the 2-year or 100 year flow or underestimates the 10-year flow (or a combination of these factors). The opposite is true if the skew values are large negative values (a large negative skew may indicate that either the 2-year or 100-year flows are underestimates or the 10-year flow is an overestimate, or a combination of these factors). If the calculated skew is outside the recommended range, careful consideration should be given to the reasonableness of the modeled 2-year, 10-year, and 100-year flows, which are what cause the calculated synthetic skew to fall outside the recommended range. Skews outside the recommended

range are an indication that the data are not conforming to typical flow frequency curve shapes, and the input data (2-year, 10-year, and 100-year modeled flows) should be reviewed carefully.

Based on a review of several examples with calculated skew values greater than 2.5, there is very little difference (less than 1%) between the final 1% plus value determined using a calculated synthetic skew value greater than  $+2.5$  and a skew value constrained to  $+2.5$ . Based on this analysis and conversations with MHFD, WWE recommends that in cases where the calculated skew falls outside the recommended range and the modeled input values are reasonable, the skew be constrained to either  $+2.5$  or  $-2.0$  (if the calculated skew is greater than  $+2.5$ , a constrained value of  $+2.5$  should be used and if the calculated skew based on modeling results is less than -2.0, a constrained skew value of -2.0 should be used).

# **4.0 CONCLUSION**

The above memorandum presents a summary of the methodology used to calculate 1% plus flows when statistical analysis of a full gage record is not possible or practical. Using methodology from USGS Bulletin 17C, Bulletin 17B, FEMA, and USACE, WWE developed a spreadsheet that uses the published equations to calculate 1% plus flows based on user inputs of: 2-year flow, 10-year flow, 100-year flow, and the assumed period of record.

WWE also conducted an evaluation of this spreadsheet and associated equations by using examples from nine locations in the Denver metropolitan area that WWE had previously used for statistical hydrologic analysis. These case studies provided several insights:

- 1. The spreadsheet calculations matched relatively closely to the calculations done by analyzing a gage's entire period of record. In other words, if appropriate 2-year, 10-year and 100-year flows are used as the input to the spreadsheet, the results are reasonably similar to results calculated when an entire gage record is available for statistical analysis.
- 2. The 1% plus flows evaluated in the nine examples that used the Bulletin 17C calculated 2 year, 10-year, and 100-year flows all exceeded the respective 200-year flows and in one case even the 500-year flows. When the synthetic statistics were used on example modeled flows, there were additional examples where the calculated 1% plus flow exceeded the 500-year flow.
- 3. Based on the 2-year, 10-year, and 100-year flows that are used as inputs, there can be a wide variation in the confidence interval associated with the 1% plus flow. For example, when there is large variation between 2-year and 100-year input flows, the 1% plus flow can be extremely high.
- 4. Based on the calibration and confidence in the CUHP methodology, WWE recommends that in most cases an assumed period of record of 30 years be used. However, a lower N value may be more appropriate for use in areas outside of MHFD or for watersheds that do not fit the assumptions of the CUHP cascading-plain approach or where confidence in model results is lower.

5. If the calculated skew coefficient falls outside the range recommended by Bulletin 17B, that is an indication that the input modeled hydrology does not conform to typical flow frequency curve patterns and the modeled hydrologic values should be reviewed for reasonableness.

# **5.0 REFERENCES**

England, John F. Jr., Cohn, Timothy, A., Faber, Beth A., et al. 2019. Guidelines for Determining Flood Flow Frequency Bulletin 17C. U.S. Geological Survey. Techniques and Methods 4-B5, Version 1.1, May 2019. Available at https://pubs.er.usgs.gov/publication/tm4B5

Federal Emergency Management Agency. 2019. Guidance for Flood Risk Analysis and Mapping, Hydrology: Rainfall-Runoff Analysis. Guidance Document 91.

Interagency Advisory Committee on Water Data. 1981. Guidelines for Determining Flood Flow Frequency. Bulletin #17B of the Hydrology Subcommittee. United States Geological Survey.

United States Army Corps of Engineers. 1996. Risk Based Analysis for Flood Damage Reduction Studies. Engineering Manual No. 1110-2-1619.

#### **Attachments:**

Attachment A: Dr. James Guo, Review of One-Percent-Plus Flow Frequency Analysis memorandum

Attachment B: STARR II, Review of MHFD One-Percent-Plus Flow Frequency Analysis memorandum for FEMA

Attachment C: 1% Plus Spreadsheet

# **Attachment A**

## **Review of One-Precent-Plus Flow Frequency Analysis**

Prepared by Dr. James C.Y. Guo, Emeritus Prof and PE Guo Engineering Inc, James.Guo@UCDenver.edu or 303-548-9119 April 6, 2022-First Draft May 23, 2022- Final

The Mile High Flood District (MHFD) retained Dr. James Guo to review the report on *One-Percent-Plus Flow Frequency Analysis* (The Report) prepared by Wright Water Engineers, Inc (WWE). This review provides a summary of comments and suggestions.

#### **Background Review**

Frequency analysis is a statistical procedure developed to establish the magnitude-recurrence relationship for a long-term hydrologic data base such as peak runoff flows or rainfall depths. The family curves of Log-Person Type III (LP III) are recommended as the underlying statistical distribution for hydro frequency analyses. The flow data are plotted as a straight line on the LP-III graphic paper (Figure 1). For the 100-yr event, the probability defined by the LP-III distribution is interpreted as:

The non-exceedance probability: *P(Q<Q100)=1-1/Tr=1/100=0.99* The exceedance probability: *P(Q≥Q100)=1/Tr=1/100=0.01* In which Prob= probability defined by LP-III distribution,  $Q=$  flow variable,  $Q100=100$ -yr peak flow, and Tr=return period in years.



 **Figure 1 Level of Confidence using 16% and 84% Limits** 

However, the nature of the hydro variable, such as stream flow, Q, is random. As a result, the value of Q100 defined by the LP III straight line represents the mean of the 100-yr peak flow, and the likelihood of the 100-yr peak flow is distributed as an error function or a normal distribution. The *one-percent-plus Flow Frequency Analysis* is a procedure to define the

confidence interval of 68% between the 16% and 84% limits. Mathematically, the probability on the normal distribution (Figure 1) is interpreted as:

The non-exceedance probability: *Prob(Q100<QU)=84%* (shaded area) The exceedance probability: *Prob(Q100≥QU)=16%* The non-exceedance probability: *Prob(Q100<QL)=16%* The exceedance probability: *Prob(Q100≥QL)=84%* The confidence interval is defined as: *Prob(OL≤O100<OU)=84%-16%=68%*, a chance of 68% for Q100 to be within QL and QU In which Prob= probability defined by normal distribution, QL= lower limit, and QU=upper limit.

## **General Comments**

The WWE's Report precisely follows the Appendix 5 in *USGS Bulletin 17B* to calculate the skewness coefficient for a synthetic data base and Appendix 7 in *USGS Bulletin 17C* to establish the 84% upper limit. Of course, this Report can be expanded to include both the upper and lower confidence limits for a specified confidence interval, depending on MHFD's needs.

The *one-percent-plus Flow Frequency Analysis* is applicable to three conditions, including (1) *Field Data* observed in the field representing the watershed historic record, (2) *Synthetic Data* generated from numerical simulations representing the watershed future condition, and (3) *Combined Data* of (1) and (2) to form a complete data base for hydro frequency analyses. The WWE's Report was prepared to focus on Synthetic Data Only. Of course, this Report can be expanded to include (1) and (3), depending on MHFD's needs.

#### **Specific Suggestions**

*Section 2.2 Example Calculation* in the WWE's Report covers all the basics in Appendix 7 in *USGS Bulletin 17C*. Due to the fact that the Normal and LP III distributions are not integrable, the WWE's report offers the Computer Model, *One-percent Plus Flow Calculator* which is well formulated to determine the 84% upper limit for the 100-yr peak flow only. The mean and standard deviation are calculated referring to the look-up table based on the calculated skewness coefficient. In practice, the user may need the look-up tables in order to evaluate the numerical accuracy. The details are presented in the appendices, including: (a) Table for Frequency Factors (Deviates) for Normal Distribution presented in Appendix II, and (b) Frequency Factors for LP III Distributions in Appendix III.

To verify the example computations in the WWE's Report, the attached Excel Spreadsheet program: *Conf-Limit* was developed. Without using any look-up tables, *Conf-Limit* presents a numerical automation algorithm for any selected confidence interval such as 84%, 90% or 95%. As demonstrated in Appendix I, there are 5 input parameters required to produce the upper and lower confidence limits for any events between 2- and 100-year return periods. These 5 inputs are: (1) the selected confidence interval, (2) 2-, 10-, and 100-yr synthetic peak flows, and (3) the length of data record.

The examples in Tables 1, 2, and 3 in the WWE's Report should include 2-, 10-, and 100-yr peak flows and the length of the stream flow record for each case. Using the automated program:

CONF-Limit in Appendix I, we may conduct the sensitivity test regarding the assumption of N=30 years of record.

## **Closing**

- (1) The WWE's Report covers the basics to determinate the magnitude of 1% Plus Synthetic Flow. The Example in Section 2.2 presents correct computations.
- (2) Although the 84% upper limit is sufficient for the concept of 1% Plus Flow, the complete confidence interval should include the 16% lower limit.
- (3) At a steam gage, we are facing two sets of peak flow-frequency curves, including (a) the stream data representing the development history of the tributary area (FEMA's preference), and (b) the synthetic data representing the future development of the tributary area (MHFD's interest).

For a floodplain delineation and master drainage study, the synthetic data predicted by the Colorado Urban Hydrograph Procedure (CUHP) should be applied to the engineering designs.

For a flood damage evaluation or forensic study, the two probability curves can be fairly combined together as:

 $Pc = P1 + P2 - P1xP2$ 

In which  $Pc =$  combined probability,  $P1 =$  probability from field data (stream gage study), and P2=probability from synthetic data (computer simulation). The combined probability fairly represents the watershed condition for flood damage assessment or risk-cost studies.

(4) The 1% plus peak flow is a preventive measure to cope with the possible impact of climate change on engineering designs. With this new policy, the engineer will have to consider two design alternatives: (a) use the 1% plus 100-yr peak flow as a conservative approach, or (2) use the 100-yr peak flow as an economic design. The MHFD may have to develop guidelines as to how to select the design alternative based on risk-cost, years of service, public safety, and traffic services.

## **Data Availability**

All data and computation sheets are available through the author: Dr. James Guo at James.Guo@UCDenver.edu

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#### **APPEND-I. Numerical Automation in CONF-LIMIT EXCEL Program**

Apply the computer model, CUHP or EPA SWMM, to the watershed to produce the 2-year flow = 50 cfs, 10-year flow = 300 cfs, and 100-year flow = 1,500 cfs. The corresponding skewness coefficient, Gs, is estimated as:

$$
G_S = -2.50 + 3.12 \frac{Log(\frac{Q_{0.01}}{Q_{0.10}})}{Log(\frac{Q_{0.10}}{Q_{0.50}})} = -2.50 + 3.12 \frac{Log(\frac{1,500}{300})}{Log(\frac{300}{50})} = 0.30
$$

For a specified return period, Tr, the exceedance probability,  $P(Q>q)$ , is defined as:

$$
P = P (Q > Q_{100}) = \frac{1}{T_r} = 0.01 \text{ when } Tr = 100 \text{ yr}
$$

The 100-yr frequency factor (or deviate),  $Z_n$ , on the Normal Distribution is calculated as (Abramowitz and Stegun, 1965):

$$
B = \sqrt{\ln\left(\frac{1}{p^2}\right)} = \sqrt{\ln\left(\frac{1}{0.01^2}\right)} = 3.03
$$
  

$$
Z_{n,0.01} = B - \frac{2.515517 + 0.802853B + 0.010328B^2}{1 + 1.432788B + 0.189269B^2 + 0.001308B^3} = 2.327 \quad \text{(=2.33 in Appendix II)}
$$

The 100-yr frequency factor, KGs,0.01, on the LPIII distribution is calculated as (Harter, 1971) (Kite, 1977):

$$
K_{GS,0.01} = \frac{2}{G_s} \left\{ \left[ \left( Z_{n,0.01} - \frac{G_s}{6} \right) \frac{G_s}{6} + 1 \right]^3 - 1 \right\} = \frac{2}{0.3} \left\{ \left[ \left( 2.327 - \frac{0.3}{6} \right) \frac{0.3}{6} + 1 \right]^3 - 1 \right\} = 2.54421
$$

Repeat the above, the 2-year frequency factor, K<sub>GS,0.5</sub>, for the LPIII distribution is:

 $K_{\text{Gs, 0.50}} = -0.04993$  (see Appendix III for comparison)

Next, the mean and standard deviation are estimated as:

$$
S_S = \frac{Log(Q_{0.01}/Q_{0.50})}{K_{G_S,0.01} - K_{G_S,0.50}} = \frac{Log(1500/50)}{2.54421 - (-0.04993)} = 0.57
$$
 (Standard Deviation)

$$
X_S = Log(Q_{0.50}) - K_{G_S,0.50}(S_S) = Log(50) - (-.04993)(0.57) = 1.73
$$
 (Mean)

Assuming that the flow data base is a 30-yr record or  $N=30$ . Set the non-exceedance probability= 0.84 or the exceedance probability is  $P=0.16$ .

$$
B = \sqrt{\ln\left(\frac{1}{p^2}\right)} = \sqrt{\ln\left(\frac{1}{0.016^2}\right)} = 2.8758
$$

$$
Z_{0.84} = B - \frac{2.515517 + 0.802853B + 0.010328B^2}{1 + 1.432788B + 0.189269B^2 + 0.001308B^3} = 1.0
$$
 (=0.995 from Appendix II)

$$
a = 1 - \frac{(Z_{0.84})^2}{2(N-1)} = 1 - \frac{(1)^2}{2(30-1)} = 0.98
$$

$$
b = (K_{GS,0.01})^2 - \frac{(Z_{0.84})^2}{N} = (2.54421)^2 - \frac{(1)^2}{30} = 6.44
$$

$$
K_{0.01,0.84}^U = \frac{K_{Gs,0.01} + \sqrt{(K_{Gs,0.01})^2 - ab}}{a} = \frac{2.54421 + \sqrt{(2.54421)^2 - (0.98)(6.44)}}{(0.98)} = 3.01
$$

$$
U_{0.01,0.84} = X_S + S_S(K_{0.01,0.84}^U) = 1.73 + 0.57(3.01) = 3.45
$$

 $1\%$  Plus Flow =  $10^{U_{0.01,0.84}} = 10^{3.45} = 2818$  cfs (the upper limit of 84% confidence)

$$
K_{0.01,0.16}^{L} = \frac{K_{GS,0.01} - \sqrt{(K_{GS,0.01})^2 - ab}}{a} = 2.1857
$$
  

$$
U_{0.01,0.16} = X_S + S_S(K_{0.01,0.16}^{L}) = 1.73 + 0.57(2.1857) = 2.9758
$$

1%  $Plus Flow = 10^{U_{0.01,0.16}} = 10^{2.9758} = 945 cfs$  the lower limit for 84% confidence

Repeat the above procedure to determine the two limits of 16% and 84% confidence for the 2- and 10-year events



Conf-Limit is the Excel Spreadsheet Program using an auto numerical procedure.



# CASE I: Confidence Interval from 16% to 84%

## Case II Confidence Interval between 5% and 95% Limits





# **APPENDIX II Frequency Factor, Z, (Deviates) for Normal Distribution**

## THE NORMAL DISTRIBUTION FUNCTION (Zn=z)

If Z has a normal distribution with mean 0 and variance 1 then, for each value of  $z$ , the table gives the value of  $\Phi(z)$ , where

 $\Phi(z) = P(Z \leq z)$ . Non-Exceedance Probability



For negative values of z use  $\Phi(-z) = 1 - \Phi(z)$ .





# Appendix III Frequency Factors (Deviates) for Pearson Type III Distribution



Exceedance Probability for Log Pearson III Distribution

# Response to MHFD's Review Comments (5/16, 5/23/2022)

#### Reviewer's Questions:

- 1. We would like to discuss a recommendation for the period of record (N) to be used for computing the synthetic frequency curve within the MHFD's boundary since it is sensitive for the confidence limits. The review memo did not elaborate the N value.
- 2. We might need a sensibility analysis to determine the recommended N for the areas within District's boundary. What is the recommended procedure for the sensitivity analysis?

**Answer:** Referring to WWE's report, the calculation example in Section II is used to demonstrate the sensitivity analysis of N value to the 84% upper limit. The value of N is involved in the calculations of two variables: a and b as:

$$
a = 1 - \frac{(Z_{0.84})^2}{2(N-1)} = 1 - \frac{(1)^2}{2(N-1)}
$$
  

$$
b = (K_{GS,0.01})^2 - \frac{(Z_{0.84})^2}{N} = (2.54421)^2 - \frac{(1)^2}{N}
$$

In practice, the minimum record length for N is 10 years and the maximum record length for N is 100 years. A sensitivity analysis is conducted for the 100-yr peak flow, Q100=1500 cfs based on the LP-III distribution. Table 1.1 summarizes the ratios of QU/Q100 for N=10, 20, 30, ….100. As expected, the ratio, QU/Q100, continually decreases toward unity as the value of N increases. It takes N>1000 year to reach QU/Q100=>1.0. Figure 1.1 is the plot of QU/Q100 varied with respect to N. It is noticed that we have an increasing return as N<30 and a decreasing return as N>30. Therefore, it is reasonable to recommend that N=30 based on the diminishing return. To generalize this observation, it may take more data sets at various locations within the MHFD's boundary.

#### **Table 1.1 Sensitivity of 84% Upper Limit to the Length of Record (QU=84% upper Limit)**





**Figure 1.1 N=30 Based on Diminishing Return** 

#### Reviewer's Questions:

3. The procedure to determine the Zc is implicit in the Bulletin 17B. And the procedure to calculate the confidence limits is not discussed in the Bulletin 17C.

**Answer:** The value of Zc depends on the confidence limits. For instance, Zc=1.0 for 84% upper limit while Zc=1.645 for 95% upper limit. Referring to Appendix II, the value of Zc can be read off the Normal Distribution Chart or determined by the step-by-step procedure illustrated in Appendix I to numerically solve for the value of Zc.

#### Reviewer's Questions:

4. The explanation for Zc is not clear about the annual-chance-of-exceedance versus the exceedance probability for a normal distribution.

#### **Answer:**

Annual-chance-of-exceedance probability is referred to the exceedance probability determined with the LP III distribution. For instance, the annual-chance-of-exceedance probability for the 100-yr peak flow is: P(Q≥Q100)=1/Tr= 1/100=0.01 in which Tr= return period in years. On the other hand the nonexceedance probability: P(Q<Q100)=1-P(Q≥Q100)=1-1/Tr= 0.99

Referring to Figure 1, the non-exceedance probability for a normal distribution is defined by the specified upper limit, the 84% upper limit means the shaded area under the normal distribution is equal to 0.84 and the corresponding frequency factor is: Z =1.0. (Noted: Z=Zc used in Appendix 5, Bulletin 17B or Z=Zn used in this report). As a result, the non-exceedance probability is defined as: P(Q100<UQ)=0.84 or the exceedance probability, P(Q100≥QU)=1-0.84=0.16.

#### Reviewer's Questions:

5. The comments provided in the memo seem to focus on the calculation and spreadsheet. We are interested in your thoughts and comments on the gage analysis comparison and high skew coefficient of the WWE's memo.

**Answer:** A case with a low or high skewness coefficient implies that there exist inliers or outliers in the hydro data series. The engineer should re-visit the data set to identify zero-flow events or/and historical events. There are several methods recommended to cope with inliers and outliers. With a proper adjustment to the local stream data base, Bulletin 17B further suggests to weigh the local skewness coefficient with the national skewness coefficient as show in Fig 1-2. Details can be found elsewhere (referring to the package: HEC-SSP https://www.hec.usace.army.mil/SOFTWARE/hec-ssp/documentation.aspx)



**Figure 1-2 National Map for Recommended Skewness Coefficients** 

# **Attachment B**



# **MEMORANDUM**



#### **Review of WWE Memorandum: One-Percent-Plus Flow Frequency Analysis**

#### **1. Overview**

The memorandum responded to the comments provided by Dr. Guo and Dr. Obeysekera, which were mainly focused on the sensitivity of the assumed sample size N (based on the assumption of rainfall-runoff model without calibration) and critical value Z<sub>c</sub>. The memorandum also pointed out the impact of very small (or zero) and extremely large flow values on skewness coefficient. In general, the equations, examples and corresponding spreadsheet are very well presented, documented, and easy-to-follow. The MHFD approach is consistent with the FEMA G&S for computing the 1-percent plus discharge using rainfallrunoff models. Additional comments and recommendations for incorporation regarding the equivalent record length estimation and 2D watershed modeling are detailed below.

#### **2. Comments and Recommendations**

The additional comments are focused on: I) sample size N for steady-state analysis, and II) the applicability to 2D (including 1D) unsteady-state analysis.

#### **I: Sample size N for steady-state analysis**

The memorandum focused on rainfall-runoff modeling without calibration. We further investigated the sensitivity of the sample size N for the 1-percent-plus frequency factor. Based on the national skewness coefficient map and the range of the skewness coefficients given in Bulletin 17B, Figure 1 graphically showed the change of the frequency factor (for 1-percent-plus discharge) with sample size N. By comparing the change of decreasing rate,  $N = 30$  proposed in the memorandum may be reasonably applied under the assumption of rainfall-runoff routing model without calibration.



Figure 1. Comparison of Frequency factor (1-percent-plus) vs. Sample size N.

In addition to the calculation example presented in section 2.2, the 1-percent-plus discharge was evaluated for nine locations (3 on Van Bibber Creek, 4 on Lena Gulch and 2 on Little Dry Creek) with gage records.

- WWE Table 1 applied gage analysis results (2-year, 10-year, 100-year) to estimate the 1 percent-plus discharge and compared with that obtained directly from HEC-SSP.
- WWE Table 3 applied the hydrology output to estimate the 1-percent-plus discharge and compared with that obtained from HEC-SSP.

#### *Recommendations:*

N = 30 may be reasonable to estimate 1-percent-plus discharge if frequency function is estimated from rainfall-runoff model without calibration. However before conducting the Floodplain delineation or proposing for future development with steady-state analysis, discharge frequency function is usually evaluated with:

- a) gage analysis, or
- b) the rainfall-runoff model that generally needs to be calibrated/validated using gage records (for gauged watersheds) or at least to be compared with the peak discharge of given frequency using the publications from USGS streamstats, FIS study or other verified resources (for ungauaged watersheds) for the existing conditions.

Following the discussion in the USACE Engineering Manual No. EM 1110-2-1619, N = 30 needs to be further evaluated as:

- a) Setting N as the record length of the gage records (WWE Table 1) to evaluate 1-percent-plus discharge with the calculation method presented in section 2.1.
- b) It is recommended to evaluate at least one ungauged watershed by setting N as the average record length of the regional study to compare with the results for N =30 as well as the verified publications.





experience with similar studies.

#### **II: Applicability to 2D (including 1D) unsteady-state analysis**

With the increasing attention and necessity to apply 2D unsteady-state analysis, the flood hydrograph (rather than only peak discharge) is needed for floodplain delineation and damage analysis. Thus, there is a need to further evaluate the possibility to incorporate 1-percent-plus peak discharge into 2D unsteadystate modeling. Under the assumption of precipitation event of a given frequency (e.g. 100-YR) resulting the flood event of same frequency, we recommend incorporating the 1-percent-peak discharge as following:

a) Estimating the 1-percent-plus flood hydrograph from the 1-percent-plus frequency storm directly. As stated in Atlas 14, the GEV distribution is applied for rainfall frequency analysis. Furthermore, Monte Carlo simulation (sample size of 1000) is applied to evaluate the uncertainty with 90% confidence interval for the storm of given frequency. Thus, it is reasonable to apply a normal distribution to quantify the uncertainty of the 1-percent frequency storm. Applying the standard normal distribution, the standard error of the 1-percent frequency storm (84%) may be computed as:

$$
S = \frac{D_{0.01 \cdot 0.95} - D_{0.01 \cdot 0.5}}{Z_{0.95}}
$$
 (1)

Where:  $D_{0.01,0.5},D_{0.01,0.95}$  represent the estimated depths of 1-percent frequency storm and its upper limit of 90% confidence interval.  $Z_{0.95} = 1.645$ .

The depth of 1-percent-plus frequency storm may then be estimated as:

$$
D_{1\%-plus} = D_{0.01,0.5} + S \tag{2}
$$

Applying 1-percent-plus frequency storm to a rainfall-runoff model, one may obtain the corresponding 1-percent-plus flood hydrograph. Additionally, it is necessary to compare the peak discharge from the computed flood hydrograph with that estimated using methodology given in section 2.1 of the memorandum.

b) Or, estimating the 1-percent-plus flood hydrograph from the 1-percent-plus discharge (computed using methodology provide in section 2.1) as follows:

- (i) Select a set of 1-percent-plus frequency storm candidates (with precipitation depth bounded by  $D_{0.01, 0.5}$  and  $D_{0.01, 0.95}$ );
- (ii) Perform rainfall-runoff analysis (e.g. HEC-HMS model) to obtain the corresponding flow hydrograph;
- (iii) Compare the peak discharge (obtained from each flow hydrograph) with the 1-percent plus peak discharge as:

$$
ARD_i = \frac{|Q_i - Q_{1percent - plus}|}{Q_{1-percent - plus}}
$$
 (3)

Where: ARD<sub>i</sub>, Q<sub>i</sub> stand for the absolute relative difference and peak discharge obtained from the i-th 1-percent-frequency storm candidate.

(iv) The storm candidate yielding smallest ARD is then chosen as the 1-percent-plus frequency storm.

It is worth noting: Even though the usual assumption is that 1% precipitation results in 1% flow, it is needed to also pay attention to the storm duration applied to the study following proper guidance (or standard).

#### *Recommendations:*

If an unsteady flow hydrograph is required for modeling in a 2D or 1D unsteady hydraulic model, it is recommended to further evaluate the 1-percent-plus discharge computed using methodology in section 2.1 of the memorandum by either approaches outlined above: a) running rainfall-runoff model with 1 percent-plus frequency storm; or b) comparing the peak discharge (from resulting flow hydrograph) with the 1-percent-plus peak discharge computed using the MHFD 1-percent-plus spreadsheet.

#### **References**

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